Mathematics

Introduction

1. In the technologically oriented and information rich society of the 21st century, our students require the ability to communicate, explore, conjecture, reason logically and use a variety of methods to solve problems. Students should be well prepared to further acquire new knowledge in this rapidly changing world.

2. Mathematics is increasingly important, as it provides a means to create, acquire, organize and apply information. Mathematics plays an important role in communicating ideas through making pictorial, graphical, symbolic, descriptive and analytical representations of numerical and mathematical ideas, and hence lays a strong foundation for students' lifelong learning. Mathematical experiences acquired in schools enable students to become mathematically literate citizens and contribute towards social prosperity.

Rationale

3. Mathematics is the foundation and supporting knowledge for many disciplines. It provides a basis to other Key Learning Areas (KLAs) for making investigations as well as a tool for analyzing data, representing findings and models with symbols, graphs and charts, and theorizing knowledge. Other KLAs also enrich students with examples of applying mathematics in real-life situations.

4. Mathematics is a core subject in the New Senior Secondary Curriculum and is a continuation of the junior secondary mathematics curriculum. It aims to furnish students with the necessary mathematical knowledge and skills to live successfully in society and to contribute to it. A distinctive feature of the New Senior Secondary Mathematics Curriculum (NSSMC) is its flexibility. It provides diversified pathways and opportunities to meet the varied needs and interests of students.

Curriculum Aims

5. The aims of the curriculum are to enable students to:

- ✧ become mathematically literate citizens and contribute towards social prosperity;
- ✧ cope confidently with the mathematics needed in their future studies, workplaces and real life;
- ✧ develop the ability to manipulate quantitative and spatial information at a higher level of sophistication;

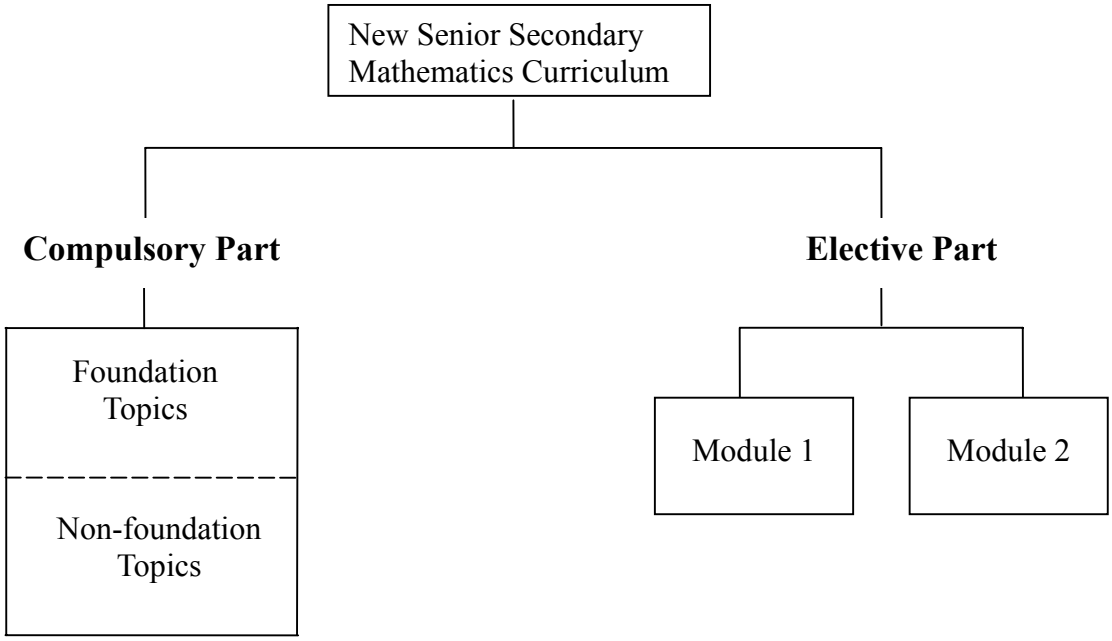
- ✧ develop the ability to communicate and express ideas clearly, concisely and logically in mathematical language;
- ✧ develop the ability to think critically and creatively, to conceptualize, inquire and reason mathematically, to use mathematics to formulate and to make use of a variety of approaches to solve real-life problems in our modern technologically oriented society; and
- ✧ develop a positive attitude towards mathematics learning and the ability to appreciate the aesthetic nature and cultural aspect of mathematics, and encourage a continuing interest in mathematics.

Curriculum Framework

(This part should be read in conjunction with the section "Curriculum Framework" of the Main Document. It should be noted that the curriculum framework suggested below is for initial consultation only. Feedback from the public will be taken into account and further details will be provided in the next stage of consultation.)

6. The NSSMC is built on the strengths of the existing mathematics curricula, and aims to provide a balanced and comprehensive learning experience, through which students develop the necessary mathematical knowledge, skills, values and attitudes for future studies, the workplace and life in society. Generic skills and real-life applications will be further developed in the curriculum to enhance students' capacity for lifelong learning; independent, critical and exploratory thinking; innovation and adaptation to change.

7. Mathematics is a core subject in the New Senior Secondary Curriculum. In order to cater for the diversified needs and interests of students, the design of NSSMC is flexible and diversified. It is composed of two parts, namely the Compulsory Part and the Elective Part. ALL students should take the Compulsory Part. The Elective Part embraces two optional modules and serves as add-on mathematics knowledge to the Compulsory Part. In brief, the proposed framework of the NSSMC can be represented diagrammatically as follows:



(Note: Students may take the Compulsory Part only, the Compulsory Part with Module 1 or the Compulsory Part with Module 2.)

8. The current design is adapted from the proposal in the report *Review of the Academic Structure of Senior Secondary Education*. Following the rationale proposed in this report, the design aims to provide students with a broad and balanced study at the senior secondary level. Instead of the provision of an elective subject in mathematics, an Elective Part is included in the NSSMC in order to offer students more flexibility and diversity in the study of mathematics.

9. The **Compulsory Part** of the NSSMC aims to provide a broad and balanced curriculum for all students. It covers the necessary mathematical knowledge and skills our students will need to equip themselves with for life in the rapidly changing information rich era. The Compulsory Part is a continuation of the mathematics curriculum at Key Stage 3. Its content is organized into three learning strands, namely Number and Algebra; Measures, Shape and Space; and Data Handling. Further applications of knowledge in these strands in real-life situations will be introduced. The Compulsory Part is designed to prepare students for lifelong learning and for future studies and the workplace. An overview of the learning targets of the 3 strands of the Compulsory Part is provided below:

Number and Algebra	Measures, Shape and Space	Data handling
Students are expected to:		
• understand the real number system; • investigate and describe relationships between quantities using algebraic symbols and relations; • generalize and describe patterns in sequences of numbers using algebraic symbols, and apply the results to solve problems; • interpret more complex algebraic relations from numerical, symbolic and graphical perspectives; • manipulate more complex algebraic expressions and relations, and apply these knowledge and skills to formulate and solve a variety of practical problems and justify the validity of results; and • apply the knowledge and skills in the Number and Algebra strand to generalize, describe and communicate mathematical ideas and solve further problems in various strands.	• use and select inductive reasoning, deductive reasoning or an analytic approach to study the properties of 2-dimensional shapes; • formulate and write geometric proofs involving 2-dimensional shapes with appropriate symbols, terminology and reasons; • inquire, describe and represent geometric knowledge in 2-dimensional space using algebraic relations; • inquire, describe and represent geometric knowledge in 2-dimensional and 3-dimensional space using trigonometric functions; and • interconnect the knowledge and skills of the Measures, Shape and Space strand and other strands, and apply them to formulate and solve 2-dimensional and 3-dimensional problems with various strategies.	• understand and compute the measures of dispersion; • select and use the measures of central tendency and dispersion to compare data sets; • investigate and judge the validity of arguments derived from the data set; • acquire basic techniques in counting; • formulate and solve further probability problems by applying simple laws; and • integrate the knowledge in statistics and probability to solve real-life problems.

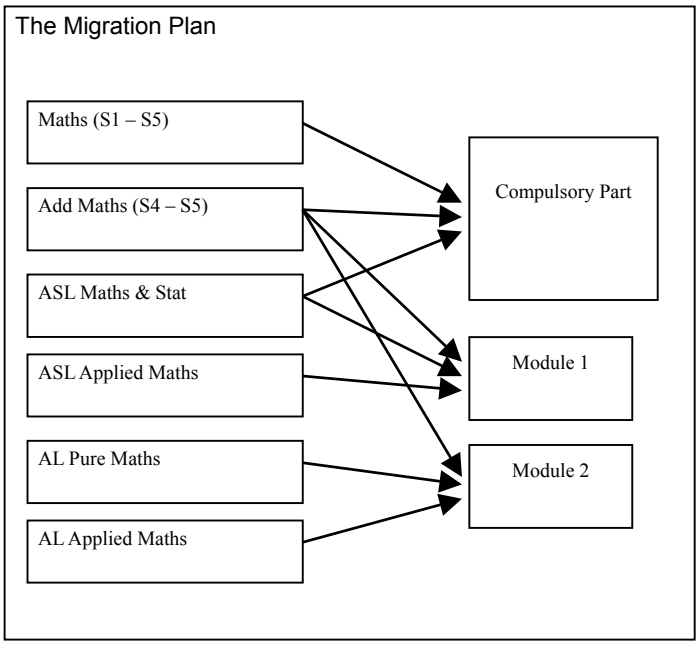
10. The proposed time allocation for the Compulsory Part ranges from 10% to 12.5% of the total lesson time, subject to the different pathways, orientations and learning pace of the students. For most students, this proposed lesson time will be sufficient to cover the Compulsory Part in 3 years.

11. To allow flexibility and leave room for the betterment of learning in the subject and to cope with students' progress, needs and interests, foundation topics and non-foundation topics are identified in the Compulsory Part. Students should have a better understanding of the foundation topics before proceeding to the learning of the non-foundation topics.

12. The **Elective Part** is designed for students who need more mathematical skills for their future careers and for those whose interests and maturity have been developed to a level that enables them to benefit from more mathematical study in different areas.

13. Two modules are offered in the Elective Part. Module 1 aims to introduce elementary knowledge and skills in the areas of Calculus, Probability and Statistics required in workplaces, daily life and future studies. Module 2 aims at extending students' mathematics horizons beyond the Compulsory Part, providing a broader scope for more advanced mathematical concepts and processes. Students are allowed to take at most one of the two modules. In other words, students may take the Compulsory Part only, the Compulsory Part with Module 1 or the Compulsory Part with Module 2. The proposed time allocation for the Compulsory Part with one module in the Elective Part is 15% of the total lesson time. It is recommended that students' achievements in the Compulsory Part and the Elective Part be reported separately.

14. The linkage between the NSSMC and the existing mathematics curricula can be summarized as follows:



The draft content of the Compulsory Part and the two modules in the Elective Part is listed in the following tables:

Compulsory Part		**Suggested lesson time (in hours)**
Topics	**Sub-topics**	
"Number and Algebra" Strand		
1. Functions and graphs	function concepts; domain and range; quadratic functions; graphical representation of a function; reading values of x satisfying $f(x)>k$, $f(x)<k$, $f(x)\geq k$ or $f(x)\leq k$ from the graph of the function; simple transformations like $kf(x)$, $f(x)+k$, $f(x+k)$, $f(kx)$; applications	15
2. More about polynomials	division of polynomials; the Remainder Theorem and the Factor Theorem	7
3. Arithmetic and geometric sequences and their summation	arithmetic and geometric sequences; series; applications	8
4. Quadratic equations in one unknown	solving quadratic equations by the factor method, formula and the graphical method; nature of roots	12
5. More about equations	solving equations (logarithmic, trigonometric, rational equations and equations involving unknown indices) reducible to quadratic equations; solving simultaneous equations with one linear and one quadratic by algebraic methods and the graphical method; location of roots of the equation $f(x)=0$ by reading the graph of $y = f(x)$	12
6. Variations	direct, inverse, joint and partial variations	10
7. Inequalities	compound linear inequalities in one unknown; quadratic inequalities in one unknown; linear inequalities in two unknowns; linear programming	18
8. Exponential and logarithmic functions	graphs and properties; real-life applications	14
"Measures, Shape and Space" Strand		
9. Qualitative treatment of locus	sketching and verbal description of the locus of points moving under a given condition (with the aids of tools or IT instruments)	5
10. Basic properties of circles	properties of angles, chords, arcs and tangents; cyclic quadrilaterals and tests for concyclic points	27
11. Coordinate treatment of simple locus problems	equations of straight lines; equations of circles; intersections of straight lines and circles	18

Compulsory Part		Suggested lesson time (in hours)
Topics	**Sub-topics**	
12. More about trigonometry	radian measures; the three trigonometric functions (sine, cosine and tangent) and their graphs; simple trigonometric equations (from 0° to 360°); sine and cosine formulas; areas of triangles; 2D and 3D problems	22
"Data Handling" Strand		
13. Measures of dispersion	box-and-whisker diagrams; standard deviation; inter-quartile range; range; properties of dispersion	16
14. Uses and abuses of statistics	sampling techniques; data collection methods (including questionnaires); study of statistical reports	8
15. More about probability	set notations (including union, intersection and complement); sample space; addition and multiplication rules; conditional probability	19
16. Permutation and combination	definition of permutation and combination; the meaning of the symbols $r!$, ${}^{n}P_{r}$ and ${}^{n}C_{r}$; applications of permutation and combination to simple problems (excluding circular permutation); applications to probability	14
"Further Applications" Learning Module		
17. Further applications	applications of mathematics learnt in various strands to more sophisticated real-life or mathematical situations	25
	Total (in 3 years):	**250**

Elective Part: Module 1		Suggested lesson time (in hours)
Topics	**Sub-topics**	
1. Exponential functions and logarithmic functions	introduction of the number e; properties and graphs of the exponential functions and logarithmic functions to any base; simple equations involving indices and logarithms; reduction of the relation $y = kx^{n}$ to a linear relation	10
2. Introduction to differentiation with applications	average rate of change; derivatives; basic rules of differentiation (addition rule, product rule, quotient rule and chain rule); differentiation of functions involving polynomial functions, exponential functions and logarithmic functions; simple applications including rate of change, maxima and minima	25

Elective Part: Module 1		Suggested lesson time (in hours)
Topics	**Sub-topics**	
3 Introduction to integration with applications	indefinite integration and definite integration involving polynomial functions and exponential functions; simple applications of definite integrals; approximation of definite integrals using the trapezoidal rule	15
4. Probability	conditional probability and independence; Bayes' Theorem	10
5. Binomial and Poisson distributions	the concept of discrete random variables; the meaning of discrete probability function; expectation and variance; the Binomial and Poisson distributions; simple applications of means and variances of Binomial and Poisson distributions (proofs are not required)	20
6. Normal distribution	elementary properties of the normal distribution; standardization of a normal variable and use of the normal table; mean and variance of the normal distribution and problems concerning a variable X, where $X \sim N(\mu, \sigma)$; normal approximation to the binomial distribution	20
7. Statistical inference: estimation and hypothesis testing	sampling distribution of the mean; estimation of the population mean using the normal distribution with a known variance; confidence interval of the population mean with a known variance; hypothesis testing about the population mean with a known variance	25
	Total (in 3 years):	**125**

Elective Part: Module 2		Suggested lesson time (in hours)
Topics	**Sub-topics**	
1. Foundation knowledge	the Principle of Mathematical Induction; the Binomial Theorem for positive integral indices; the six trigonometric functions and the related trigonometric formulas, exponential functions and logarithm functions; use of notations of inverse trigonometric functions; radian measure; arc length and area of sector; algebraic fractions and partial fractions	28

Elective Part: Module 2		Suggested lesson time (in hours)
Topics	**Sub-topics**	
2. Limits and differentiation	intuitive concept of limits of functions; derivatives; basic rules of differentiation (addition rule, product rule, quotient rule and chain rule); differentiation of functions involved simple algebraic functions, trigonometric functions, exponential functions and logarithmic functions; differentiation of implicit functions; simple applications including rate of change, tangents and normals to a curve, maxima and minima, and sketching of simple curves; second derivatives	30
3. Integration	indefinite integration as the reverse process of differentiation; fundamental integration formulas; indefinite integrals of functions involved simple algebraic functions, trigonometric functions and exponential functions; definite integrals of functions involved simple algebraic functions, trigonometric functions and exponential functions; simple techniques of integration including the method of substitution and integration by parts; simple applications to plane areas and volumes of solids of revolution about the x-axis or the y-axis	30
4. Vectors in $\mathbf{R}^2$ and $\mathbf{R}^3$	definitions of vectors and scalars; addition and subtraction of vectors; resolution of vectors in the rectangular coordinate system; scalar (dot) product and vector (cross) product; simple applications of vectors in geometry	20
5. Matrices	matrices and their operations; square matrices of order 2 and 3; inverses of square matrices of order 2 and 3 and the application to systems of linear equations; applications of 2×2 matrices in coordinate transformations (linear transformations): reflection, rotation, enlargement, translation and shear	18
	Total (in 3 years):	**126**

Learning and Teaching

15. In designing learning and teaching activities for the NSSMC, the following principles should be noted:

- ✧ The main concern is to help students learn to learn

- ✧ All students can learn, but at different speeds
- ✧ A learner-focused approach should be adopted
- ✧ Information technology, when used appropriately, increases the effectiveness of learning and teaching

16. To assist teachers to adjust the Compulsory Part to meet the needs of their students, foundation topics are identified. All students should strive to master these. Schools may select the non-foundation topics and adjust the depth of treatment of these to cater for the individual needs of their students.

Assessment

(This part should be read in conjunction with the section "Assessment" of the Main Document.)

17. Assessment is the practice of collecting evidence of student learning. The aims are to improve learning and teaching as well as to recognize the achievement of students. It is an integral part of the learning-teaching cycle. The assessment design will align with curriculum aims, design and learning processes of the subject. Since mathematics is not simply a collection of isolated facts and information, but a field of inquiry that is built on a network of interrelated ideas, the assessment of the NSSMC should focus on students' abilities to communicate, explore, conjecture, reason logically and apply a variety of mathematical methods to solve problems. The essence of assessment is to recognize student performance with respect to the Learning Targets and Learning Objectives of the NSSMC.

18. **Internal assessment** refers to the assessment practices that schools employ as part of the learning and teaching strategies during the three year study in Mathematics. Internal assessment embraces a wide range of assessment practices that is administered in schools as part of the learning and teaching process. During the three years of senior secondary education, schools should assess students formatively, in order to collect continuous feedback on student achievements vis-à-vis the set objectives, so that professional judgments can be made about student progress, and about how to improve the learning and teaching process. Information from formative assessment will help motivate student learning, and help teachers to find ways of promoting effective learning and teaching.

19. Public Assessment of Mathematics leads to a qualification in the subject to be offered by the Hong Kong Examinations and Assessment Authority. In the public assessment of Mathematics, a standards-referenced approach will be adopted for grading and reporting student performance. The purpose of this approach is to recognize what each student can

do in the subject at the end of the 3-year senior secondary education. Each student's performance will be matched against a set of performance standards, rather than compared to the performance of other students. It makes the implicit standards explicit by providing specific indication of student performance. Descriptors will be provided for the set of standards at a later stage. Public assessment of the NSSMC will comprise two components: School-based Assessment (SBA) and a Written Examination. Students will be assessed continuously through SBA. The Written Examination will consist of various types of items to assess student performance in a broad range of skills and abilities.

20. Public assessment will include an SBA component. As an initial proposal, the component will take up 20% of the total weighting. The merits of adopting SBA are as follows:

(a) SBA is able to provide a more valid assessment than external assessment, since it can cover a more extensive range of learning outcomes through introducing a wider range of assessment practices than is possible in external written examinations.

(b) SBA enables the sustained work of students to be assessed. It provides a more comprehensive picture of student performance throughout the period of study rather than their performance in a one-off examination alone.

21. It should be noted that SBA is not an "add-on" element in the curriculum. Assessing student performance through practices such as class discussion and class observation is a normal in-class and out-of-class activity. The modes of SBA selected in the NSSMC will be appropriate to the learning objectives and processes to be assessed. The design and implementation of SBA should aim to avoid unduly increasing the workload of both teachers and students.

Supporting Measures

22. The following learning and teaching materials will be developed to support the implementation of the NSSMC:

(a) A single subject curriculum and assessment guide will be published jointly by the Curriculum Development Council and the Hong Kong Examinations and Assessment Authority to provide various stakeholders with information on the rationale, aims, curriculum framework, learning targets, learning objectives, learning and teaching strategies, and internal and public assessment of the NSSMC.

(b) Exemplar booklets aiming to provide teachers with resource materials for teaching the NSSMC will be developed. These booklets will contain exemplars, examples of good practice and suggested approaches in teaching the Compulsory Part and the two modules in the Elective Part. Teachers may adjust these resources or to develop their own learning and teaching materials to suit their individual needs.

23. It is anticipated that textbooks on the NSSMC will also be available when the NSSMC is implemented.

24. Professional development programmes and seminars will be organized before the implementation of the NSSMC to help teachers develop a better understanding of the curriculum, followed by professional development programmes to focus on knowledge updating, the pedagogy of the subject, SBA and standards-referenced assessment. The courses will not only be organized for panel heads/coordinators, but also for all teachers teaching the NSSMC. The courses on the understanding and planning of the NSSMC will be conducted as early as possible while those on SBA and assessment will be organized for teachers shortly before the implementation. The courses on knowledge updating and the pedagogy of the NSSMC will be planned and organized once the content of the NSSMC is finalized.

25. Tertiary institutions and professional bodies will be invited to offer assistance to plan, organize and run the teacher development courses as well as to provide inputs to develop the exemplar booklets.

26. Focus group discussions with teachers will be held to solicit views of frontline teachers on the implementation of the curriculum, thus enabling curriculum developers to be aware of potential problems as early as possible. On the basis of teacher feedback, appropriate adjustments will be made to the NSSMC, wherever possible, to avoid or overcome the problems. In addition, teacher networks and learning communities will be formed to discuss various issues and aspects related to the implementation of the curriculum.